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MA110 - Engineering Mathematics-1
Problem Sheet - 4

Directional Derivatives and Gradient Vectors

1. What is the derivative of a function $f(x,y)$ at a point P_0 in the direction of a unit vector $\mathbf{u}$? What rate does it describe? What geometric interpretation does it have? Give examples.

2. What is the gradient vector of a differentiable function $f(x,y)$? How is it related to the functions directional derivatives? State the analogous results for functions of three independent variables.

3. Find the derivative of the function at P_0 in the direction of $\mathbf{A}$.

   (a) $f(x,y) = 2xy - 3y^2$, $P_0(5,5)$, $\mathbf{A} = \mathbf{4i} + \mathbf{3j}$
   (b) $h(x,y) = \tan^{-1}(y/x) + \sqrt{3}\sin^{-1}(xy/2)$, $P_0(1,1)$, $\mathbf{A} = \mathbf{3i} - \mathbf{2j}$
   (c) $g(x,y,z) = 3e^x \cos yz$, $P_0(0,0,0)$, $\mathbf{A} = \mathbf{2i} + \mathbf{j} - \mathbf{2k}$
   (d) $f(x,y,z) = xy + yz + zx$, $P_0(1,-1,2)$, $\mathbf{A} = \mathbf{3i} + \mathbf{6j} - \mathbf{2k}$

4. Find the directions in which the functions increase and decrease most rapidly at P_0. Then find the derivatives of the functions in these directions.

   (a) $f(x,y) = x^2 + xy + y^2$, $P_0(-1,1)$
   (b) $f(x,y,z) = \ln xy + \ln yz + \ln xz$, $P_0(1,1,1)$

5. Find the directions in which the functions increase and decrease most rapidly at P_0. Then find the derivatives of the functions in these directions. Also find the dertivative of f at P_0 in the direction of the vector $\mathbf{v}$.

   (a) $f(x,y) = x^2e^{-2y}$, $P_0(1,0)$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$
   (b) $f(x,y,z) = \ln(2x + 3y + 6z)$, $P_0(-1,-1,1)$, $\mathbf{v} = \mathbf{2i} + \mathbf{3j} + \mathbf{6k}$

6. In what direction is the derivative of $f(x,y) = xy + y^2$ at $P(3,2)$ equal to zero?

7. Is there a direction $\mathbf{u}$ in which the rate of change of $f(x,y) = x^2 - 3xy + 4y^2$ at $P(1,2)$ equals 14? Give reasons for your answer.

8. Is there a direction $\mathbf{u}$ in which the rate of change of the temperature function $T(x,y,z) = 2xy - yz$ (temperature in degrees Celsius, distance in feet) at $P(1,-1,1)$ is -3 deg.Cel/ft ? Give reasons for your answer.

9. The derivative of $f(x,y,z)$ at a point P is greatest in the direction of $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$. In this direction, the value of the derivative is $2\sqrt{3}$.

   (a) What is $\bigtriangledown f$ at P ? Give reasons for your answer.
   (b) What is the derivative of f at P in the direction of $\mathbf{i} + \mathbf{j}$?

10. What is the largest value that the directional derivative of $f(x,y,z) = xyz$ can have at the point $(1,1,1)$?

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